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DEPAF—A Computer Model for Design and Performance Analysis of Furnaces

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This report covers the development of a computer simulation program DEPAF (DEsign and Performance Analysis of Furnaces) for residential fossil-fuel-fired furnaces. DEPAF is based upon an analytical model which accounts for cyclic (on-and-off) operation of furnace burner and blower. Transmission of heat at on-cycle uses the theory of radiative and convective heat transfer; transmission of heat at off-cycle uses the theories of turbulent and free convective heat transfer. Confidence in DEPAF was established by the use of available experimental data on a gas-fired forced-warm-air furnace. While the theory of transient heat transfer in combustion is complex in nature, theoretical results based upon quasi-steady-state analysis are in excellent agreement with experiments. If the building heat loss is known, DEPAF can be used to calculate the annual performance and operating cost for residential heating systems with furnaces. Examples are given to illustrate applications of DEPAF to examine quantitatively the effect of design and operating variables on annual performance and operating costs of residential forced-warm-air furnaces. It was found that considerable savings in fuel and operating costs can often be achieved by performing certain modifications to existing furnaces.

***The Author is also known as S. W. Chi.**

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ABSTRACT

This report covers the development of a computer simulation program DEPAF (DEsign and Performance Analysis of Furnaces) for residential fossil-fuel-fired furnaces. DEPAF is based upon an analytical model which accounts for cyclic (on-and-off) operation of furnace burner and blower. Transmission of heat at on-cycle uses the theory of radiative and convective heat transfer; transmission of heat at off-cycle uses the theories of turbulent and free convective heat transfer. Confidence in DEPAF was established by the use of available experimental data on a gas-fired forced-warm-air furnace.

While the theory of transient heat transfer in combustion is complex in nature, theoretical results based upon quasi-steady-state analysis are in excellent agreement with experiments. If the building heat loss is known, DEPAF can be used to calculate the annual performance and operating cost for residential heating systems with furnaces. Examples are given to illustrate applications of DEPAF to examine quantitatively the effect of design and operating variables on annual performance and operating costs of residential forced-warm-air furnaces. It was found that considerable savings in fuel and operating costs can often be achieved by performing certain modifications to existing furnaces.

NOMENCLATURE

A_b = heat exchanger area

$\overline{AB}$ = effective radiative flux area

B = net heating value of a fuel

C = wall heat capacity

C_b = heat capacity of heat exchanger wall

C_{pa} = specific heat of air

C_{pf} = specific heat of fuel

C_{pp} = specific heat of combustion products

$\dot{C}$ = heat capacity rate

$\dot{C}_a$ = heat capacity rate of combustion products or draft air

$\dot{C}_{a,f}$ = heat capacity rate of combustion products at steady state

$\dot{C}_{a,off}$ = heat capacity rate of draft air at off cycle

$\dot{C}_{a,on}$ = heat capacity rate of combustion products at on cycle

$\dot{C}_b$ = heat capacity rate of heat exchanger wall, which is equal to zero

$\dot{C}_c$ = heat capacity rate of circulating air

$\dot{C}_{c,f}$ = heat capacity rate of circulating air at steady state

$\dot{C}_{c,off}$ = heat capacity rate of circulating air while blower is off

$\dot{C}_{c,on}$ = heat capacity rate of circulating air while blower is on

D = Kg of unburned carbon per Kg of fuel fired

G_{ab} = total heat conductance coefficient between combustion products (or draft air) and heat exchanger wall

$G_{ab,c}$ = convective component of G_{ab} at on cycle

$G_{ab,f}$ = G_{ab} at steady state

$G_{ab,off}$ = G_{ab} at on cycle

$G_{ab,on}$ = G_{ab} at on cycle

$G_{ab,r}$ = radiative component of G_{ab} at on cycle

G_{bc} = total heat conductance coefficient between circulating air and heat exchanger wall

$G_{bc,f}$ = G_{bc} at steady state

$G_{bc,off}$ = G_{bc} while blower is off

$G_{bc,on}$ = G_{bc} while blower is on

FR = furnace fuel firing rate at steady state

HHV = higher heating value of fuel

K = mass ratio of air to fuel at steady state

L_b = heat exchanger height

L_e = mean beam length for radiative heat transfer

$\dot{m}_a$ = mass flow rate of combustion products or draft air

$\dot{m}_{a,f}$ = mass flow rate of combustion products at steady state

$\dot{m}_{a,off}$ = mass flow rate of draft air at off cycle

$\dot{m}_{a,on}$ = mass flow rate of combustion products at on cycle

M = Kg of moisture formed per Kg of fuel fired

N = Kg of CO formed per Kg of fuel fired

P_{ch} = sum of partial pressures of CO_2 and H_2O in combustion products

t = time

T_a = temperature of combustion products or draft air

$T_{a,f}$ = temperature of combustion products at steady state

$T_{a,off}$ = temperature of draft air at off cycle

$T_{a,cn}$ = temperature of combustion products at on cycle

T_b = temperature of heat exchanger wall

$T_{b,off}$ = temperature of heat exchanger wall while blower is off

T_c = temperature of circulating air

$T_{c,off}$ = temperature of circulating air while blower is off

T_o = temperature of furnace room or outdoor air as defined in text

T_r = furnace room temperature

T_A, T_B, T_C, T_D, T_E = Values of temperature at modal points shown in figure 2

T'_B, T'_C, T'_E = values of T_B, T_C, and T_E at a time Δt later

Δt = step increment of t

$\Delta T_{Lm,ac}$ = logarithmic temperature difference between combustion products and circulating air at steady state

$\Delta T_{Lm,bc}$ = logarithmic temperature difference between heat exchanger wall and circulating air at steady state

x = position of heat exchanger wall measured from gas inlet end

Δx = step increment of x

σ = Stefan-Boltzmann constant

ε_a = emissivity of combustion products

ε_b = emissivity of heat exchanger wall

INTRODUCTION

In the past, building heating/cooling equipment performance ratings were based upon steady-state test methods. This often led to dispute on the annual or seasonal performance of different heating/cooling equipment. This dispute has been intensified by the energy crisis and numerous field studies have been undertaken. However, there is at the present time a lack of confidence in extrapolating experimental data from one building, one heating/cooling system and one weather pattern to other buildings, systems and climate. In addition, there is also much uncertainty as to the effectiveness of the various energy saving proposals. This has led to the development of complex computer programs at NBS and several other laboratories in evaluating and ultimately reducing annual energy consumption in homes and other buildings. This report covers the development of a computer simulation program DEPAF (DEsign and Performance Analysis of Furnaces) for residential fossil-fuel-fired furnaces.

Currently, several analytical methodologies for evaluating the annual performance of fossil-fuel-fired furnaces and boilers are under investigation. They include:
1) empirical correlation between steady-state and seasonal efficiency [1][1];
2) a method based upon a finite number of improvement factors characteristic to certain equipment features [1];
3) simplified computer analysis based on stack losses using stack-gas temperature and flow data [2]; and 4) computer simulation based upon heat-transfer calculations [3,4].
Simulation program DEPAF discussed here is of the last category, as is the last mentioned reference [4] by this same author. Reference 4 documents a simulation program (DEPAB) for Design and Performance Analysis of Boilers; this paper (DEPAF) is concerned with residential furnaces.

While charts are being generated, using DEPAB and DEPAF, to simplify calculations of the residential fossil-fuel-fired heating systems without the use of a computer, this paper is devoted to a description of the furnace simulation program DEPAF. For this purpose, an analytical furnace model will first be presented. Then, differential equations describing operation of the furnace will be derived. Finally, development of the numerical procedure will be described. It will be seen that results of DEPAF are in excellent agreement with the available laboratory data on a gas-fired furnace and that the present program can facilitate selection of furnace operating and design variables to achieve annual fuel savings.

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[1] Numbers in brackets designate references at end of this paper.

FURNACE MODEL

In order to study the seasonal performance of furnaces, an analytical model, which is both an adequate representation of the physical system and capable of reasonably simple mathematical description, must be established. Figure 1 shows schematically such a model, which contains all essential elements of a residential furnace system under consideration. A blower moves circulating air through the air side of a heat exchanger. A burner supplies the required fuel. Combustion products (or draft air when the burner is off) flow through the gas side of the heat exchanger. A stack draws combustion products (or draft air) and relief air to its outlet. When the heat demand by the residence is less than the steady-state full-load output, the furnace will operate in a cyclic manner. Cyclic operation of the furnace can conveniently be divided into four successive time periods, namely: 1) burner on and blower off, 2) burner on and blower on, 3) burner off and blower on, and 4) burner off and blower off.

A room thermostat calling for heat starts off period 1. Heat is transferred from the combustion products to the heat exchanger. Part of this heat is stored in the heat exchanger wall and the balance transmitted to the circulating air. As the blower is off, a low flow rate of circulating air is induced by free convection, and the temperature of circulating air leaving the heat exchanger rises rapidly. When the circulating air leaving the heat exchanger reaches a certain set-point temperature, the blower comes on and period 2 begins. During this period, heat is continuously transferred from the combustion product to the heat exchanger wall and then to the circulating air. Circulating air delivers heat at a high rate to the desired locations in the residence. When heat demand for the residence is satisfied, the room thermostat signals a control to shut off the burner; period 3 begins. Draft air at the gas side of the heat exchanger and circulating air at the air side are then heated by the hot heat exchanger wall. As the blower is on, circulating air flows at high rate and its temperature drops rapidly during this period. Finally, the blower is turned off, and the circulating air flow rate is drastically reduced. The low flow is, however, in heat exchange with the hot heat exchanger wall, which causes a momentary rise in temperature of circulating air leaving the heat exchanger. It reaches a temperature peak and then continues to drop until the burner comes on again at the call for heat from the room thermostat. The cycles are therefore repeated, when the furnace is only partially loaded.

MATHEMATICAL FORMULATION AND COMPUTER PROGRAM

For computer simulation of the above-described cyclic operation of furnaces, governing equations based upon the furnace model pictured in Figure 1 are derived below. It is convenient for description to divide the derivation into three parts: 1) derivation of the time-dependent differential equation for each of the combustion products (or draft-air), heat exchanger wall and circulating air as shown in Figure 1; 2) derivation of initial and boundary conditions for the problem; and 3) evaluation of coefficients for the differential equations.

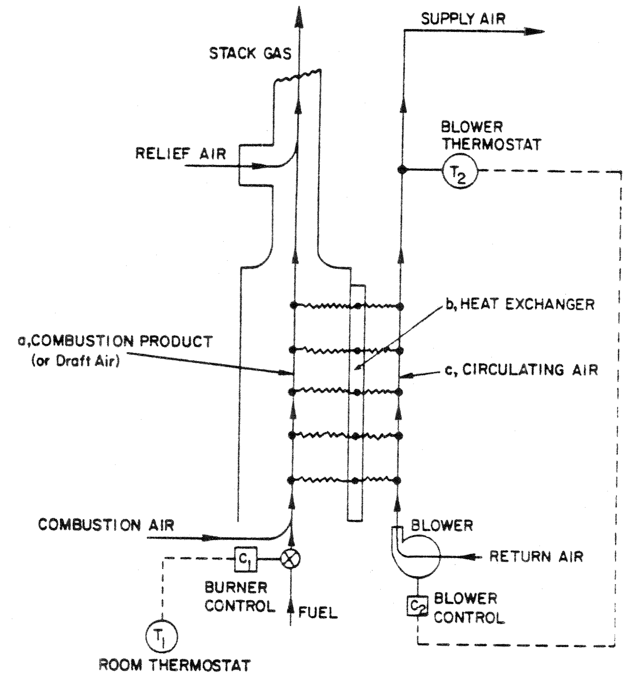

Figure 1. Schematic of a Furnace Model

Differential Equations

It can reasonably be assumed that: 1) temperatures T for the gas, wall and air are each functions of time t and distance x measured in the direction of gas flow, T = T(x,t); 2) the conductive heat transfer in x-direction and heat capacity of gas or air can be neglected. Based upon these two idealizations, the differential equations relating the system temperatures may be derived from heat balance and heat transfer considerations applied to each differential element of the system. The resultant three coupled differential equations expressing temperatures, T_a, T_b and T_c (i.e., temperatures of combustion products or draft air, heat-exchanger wall and circulating air, respectively), as functions of x and t are:

$$G_{ab}(T_b-T_a) = \dot{C}_a L_b \frac{\partial T_a}{\partial x}$$

$$G_{ab}(T_a-T_b)+G_{bc}(T_c-T_b) = C_b \frac{\partial T_b}{\partial t}$$

$$G_{bc}(T_b-T_c) = \dot{C}_c L_b \frac{\partial T_c}{\partial x} \tag{1}$$

In the above equation 1 set, G is the conductance between gas or air and its neighboring wall, C the wall heat capacity, $\dot{C}$ the gas or air heat capacity rate, and L_b the height of the furnace heat exchanger. Looking at these equations it may be noted that the left-hand terms came from heat transmission between neighboring gas or air and the heat exchanger wall; the right-hand terms of the first and third equations came from the excess of outflow over inflow thermal energy in the gas or air streams; and the right-hand term of the second equation was from the increase in thermal energy storage in solid wall.

Solutions of the equation 1 set are dependent upon initial and boundary conditions of the problem and the coefficients for these equations. They are discussed below.

Initial and Boundary Conditions

In general, performance of a furnace at cyclic operation is of interest. At the start, temperature of the system can be set at the furnace-room temperature. With cyclic variation of gas temperature entering the furnace (i.e., alternately that of the combustion products and the draft air passing through the furnace heat exchanger), successive "steady-state cycles" are established after several initial transient cycles. The cyclic boundary condition $T_a(o,t)$ is the temperature of the combustion products during the on cycle and the temperature of the draft air (at furnace room temperature in the absence of pilot flame or at temperature resulting from the combustion of pilot fuel with the draft air) during the off cycle.

During the on cycle the combustion products are heated from the furnace room temperature T_r to $T_a(o,t)$ by net heating value B(J/Kg) of the fuel.

$$B = H - (2.44 \times 10^6) M - (1.022 \times 10^7) N - (3.30 \times 10^7) D \quad (2)$$

where H is the higher heating value of the fuel, and M, N and D are kilograms of moisture, CO and unburned C, respectively, in the combustion products per Kg of fuel fired. Equating this net heat produced at firing to the sum of sensible heats in fuel, air and combustion products results in:

$$T_a(o,t) = 298 + \frac{B - (C_{pf} + KC_{pa})(298 - T_r)}{C_{pp}(1 + K)} \quad (3)$$

where C_{pa}, C_{pf}, and C_{pp} are specific heats at constant pressure for air, fuel and combustion products, respectively; T_r is the furnace room temperature and K is the air fuel ratio. Values of specific heats of gases of various compositions and temperatures may be calculated by the correlation equations [5].

Although equation 3 has been derived from on-cycle burning of the fuel, it can also be used to calculate the off-cycle gas temperature at inlet if the air fuel ratio K during off-cycle is calculated from the draft air flow rate and the fuel supply rate to the burner pilot. Therefore equations 2 and 3 constitute the boundary condition of the equation 1 set.

Coefficients

It can be seen in the equation 1 set that coefficients for these equations are dependent upon the conductance G, capacity C, and capacity rate $\dot{C}$. Value of $\dot{C}_b$ for the solid wall is zero, and value of C_b for the solid can be calculated in a straightforward manner from weight of the solid wall and material properties of the solid wall. However, temperatures and flow rates of gas and air at part load change repeatedly in response to the on and off of the burner and the blower; and consequently the gas and air conductances G_{ab} and G_{bc} and capacity rates $\dot{C}_a$ and $\dot{C}_c$ vary continuously with respect to time. Gas and air conductance and capacity rate are the main topics of this sub-section.

At full load, the burner and blower are in continuous operation. Gas flow rate at the gas side of the heat exchanger $\dot{m}_{a,f}$ can be calculated by the equation:

$$\dot{m}_{a,f} = \frac{(1 + K)\ FR}{H} \quad (4)$$

where K is the air-fuel mass ratio at firing, FR the furnace firing rate (J/h) and H the higher heating value of the fuel (J/Kg). Knowing the gas flow rate, the gas capacity rate at full load can be calculated simply by the equation:

$$\dot{C}_{a,f} = C_{pp}\dot{m}_{a,f} \quad (5)$$

Although the gas- and air-side conductances for the furnace heat exchanger of varied geometry appear difficult to generalize from the present knowledge of flame, gas radiation and gas convection at high temperature, they can be empirically determined in this manner: with gas inlet temperature $T_a(o,t)$ at full load calculated by equation 3, and the gas temperature $T_a(L_b,t)$ at outlet from the furnace empirically determined, the logarithmic mean temperature difference between the combustion products and circulating air $\Delta T_{Lm,ac}$ can then be calculated:

$$\Delta T_{Lm,ac} = \frac{[T_a(o,t) - T_c(o,t)] - [T_a(L_b,t) - T_c(L_b,t)]}{\ln \dfrac{T_a(o,t) - T_c(o,t)}{T_a(L_b,t) - T_c(L_b,t)}} \quad (6)$$

where $T_c(o,t)$ and $T_c(L_b,t)$ are the circulating air temperatures at the inlet to and outlet from the furnace heat exchanger, respectively. The overall conductance G can now be calculated simply by the equation:

$$G_{ac,f} = \frac{C_{pp}\dot{m}_{a,f}[T_a(o,t) - T_a(L_b,t)]}{\Delta T_{Lm,ac}} \quad (7)$$

Similarly, if temperatures, $T_b(o,t)$ and $T_b(L_b,t)$, of the heat exchanger wall at full load are measured, the logarithmic temperature differences between gas and heat-exchanger wall $\Delta T_{Lm,ab}$ and between heat-exchanger wall and circulating air $\Delta T_{Lm,bc}$ can be calculated by an equation identical to equation 6. With $G_{ac,f}$, $\Delta T_{Lm,ab}$ and $\Delta T_{Lm,bc}$ determined, $G_{ab,f}$ and $G_{bc,f}$ at full load can be calculated by equations:

$$G_{ab,f} = \frac{G_{ac,f}\ \Delta T_{Lm,ac}}{\Delta T_{Lm,ab}} \quad (8)$$

$$G_{bc,f} = \frac{G_{ac,f}\ \Delta T_{Lm,ac}}{\Delta T_{Lm,bc}} \quad (9)$$

and capacity rate of the circulating air at full load can be determined, considering steady-state heat transfer from combustion products to circulating air, i.e.:

$$\dot{C}_{c,f} = \dot{C}_{a,f}\ \frac{T_a(o,t) - T_a(L_b,t)}{T_c(o,t) - T_c(L_b,t)} \quad (10)$$

Gas and air capacity rates and conductances at full load have been determined in the previous paragraph. In a quasi-steady state, gas capacity rate and conductance ($\dot{C}_{a,on}$ and $G_{ab,on}$) during the burner on cycle are equal to those ($\dot{C}_{a,f}$ and $G_{a,f}$) at full load; and air capacity rate and conductances ($\dot{C}_{b,on}$ and $G_{b,on}$) during the blower on-cycle are equal to those ($\dot{C}_{b,f}$ and $G_{bc,f}$) at full load. During the time periods when the burner or the blower is off, cyclic variation of gas and air flow and heat transfer rates constructing a staggered network in the three media (i.e., gas, heat exchanger and circulating air) as shown in Figure 2. If the distances between neighboring nodes Δx and the successive time interval Δt are sufficiently small, the equation 1 set can be expressed accurately by the following equations:

$$T'_B = \frac{2\dot{C}_a L_b - G_{ab}\Delta x}{2\dot{C}_a L_b + G_{ab}\Delta x} T_A + \frac{2G_{ab}\Delta x}{2\dot{C}_a L_b + G_{ab}\Delta x} T'_C$$

$$T'_C = \frac{G_{ab}\Delta t}{2C_b}(T_A + T_B) + \frac{C_b - (G_{ab} + G_{bc})\Delta t}{C_b} T_C + \frac{G_{bc}\Delta t}{2C_b}(T_D + T_E)$$

$$T'_E = \frac{2G_{bc}\Delta_x}{2\dot{C}_c L_b + G_{bc}\Delta_x} T'_C + \frac{2\dot{C}_c L_b - G_{bc}\Delta x}{2\dot{C}_c L_b + G_{bc}\Delta x} T_D \quad (21)$$

where T_A through T_E are temperatures at the five nodes shown in Figure 2 at time t, and "prime" indicates the temperature at time $t+\Delta t$. From these three equations one can solve for system temperatures at time $(t+\Delta t)$ using known temperatures at time t. For the three equations of the equation 21 set to be stable, all coefficients must be positive. It leads to the requirements: 1) Δx is less than $2\dot{C}_a L_b/G_{ab}$ or $2\dot{C}_b L_b/G_{bc}$, whichever is smaller, and 2) Δt is less than $C_b/(G_{ab}+G_{bc})$.

After a network of nodel points has been constructed inside a furnace model (see Figure 3) equation 21 set can be applied to all nodes. Starting from t=0, a complete time history of system temperatures at all nodes can be evaluated. Time history of system mass flow rates can then be evaluated using equations 4, 10, 11 and 20. Also the stack gas flow rate can be calculated using equation 11, with $T_{a,off}$ interpreted as the stack gas temperature, which can be determined from adiabatic mixing of the relief air and furnace gas leaving the heat exchanger. From the time history of the mass flow rates and temperatures, performance of the furnace at different outdoor temperature and cycling conditions can be evaluated in a straightforward manner.

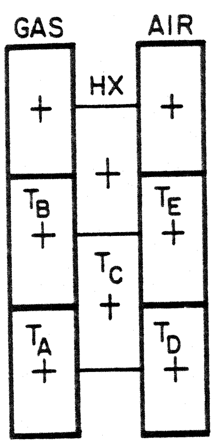

Figure 2. Elements of Staggered Nodes

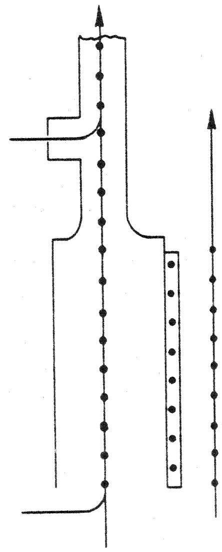

Figure 3. Schematic of Layout of Network of Nodel Points

A computer program DEPAF for design and performance analysis of furnaces has been written in accordance with the mathematical formulation described above. Figure 4 shows a flow chart for the program. It starts at reading in the fuel composition, furnace size, and structure loads at different outdoor temperatures. Four different cases, several incorporating energy saving measures, are programmed. These cases include: 1) normal mode of operation, 2) operation with increased or reduced excess air, 3) operation with modulated firing rate, 4) operation with off-cycle draft and/or relief air control, and 5) operation with preheating of combustion air by flue gas. In each specified case or combination of cases, the setpoints for room and blower thermostat can also be specified at desired values. A detailed time history of temperatures and flow rates of flue gas, stack gas and circulating air is fed to a tape unit. These data may be examined at a later time, when detailed transient and cyclic characteristics of a furnace are required to be known - e.g., in the design of control systems. Heat balance, i.e. heat input, structure load, and losses due to flue gas latent heat, flue gas sensible heat, additional structure infiltration for makeup of combustion, draft and relief air are printed out, together with the furnace efficiency at different load and weather conditions. Among these output values, furnace efficiency at part load can be used for the calculation of annual fuel consumption and operating cost for the furnace system.

Results and Discussion

In order to establish confidence in the program, the computer simulation results are first compared with experiments. The comparison is followed by an examination, using DEPAF, of the effectiveness of several proposed energy-saving measures on annual performance and operating costs of a furnace.

Comparison with Experiments

A furnace test is set up in the National Bureau of Standards mechanical equipment laboratories. Some preliminary data have been made available to the author. Figure 5 shows a comparison between the predicted and measured part-load efficiency η_ℓ for a are described below.

The capacity rate at off cycle $\dot{C}_{a,off}$ can be obtained from the relation of the gas flow rate at off cycle to that at full load. The gas flow rate in a furnace is dependent upon the difference between the hydrostatic pressures of the ambient air and the gas in the furnace and stack. The hydrostatic pressure difference [6] is proportional to $(1/T_o - 1/T_a)$ where T_o and T_a are respectively the atmospheric air and mean gas temperature of gas in the furnace and/or stack; and the flow resistance in turbulent flow [7] is proportional to $(\dot{m}_a^{1.8} \times T_a^{1.15})$ where $\dot{m}_a$ and T_a are respectively the gas flow rate and mean gas temperature. It follows that the draft air flow rate, while the burner is off, is related to the full-load gas flow rate by the equation:

$$\frac{\dot{m}_{a,off}}{\dot{m}_{a,f}} = D_f \left(\frac{T_{a,off} - T_o}{T_{a,f} - T_o}\right)^{0.56} \left(\frac{T_{a,f}}{T_{a,off}}\right)^{1.19} \tag{11}$$

where D_f is a draft constant whose value is on the order of unity for natural draft burners and on the order of 0.4 for forced draft burners, and T_a and T_o are the furnace gas and furnace room temperatures or the stack and outdoor air temperatures, dependent upon whether generous relief air opening is provided or not. With $\dot{m}_{a,f}$ calculated by equation 4 and T_a obtained from the equation 1 set, equation 11 can be used to calculate $\dot{m}_{a,off}$ during the off cycle and subsequently the off-cycle gas capacity rate can be obtained from:

$$\dot{C}_{a,off} = C_{pp,off}\dot{m}_{a,off} \tag{12}$$

Gas conductance at off cycle can also be scaled down from its value at full load. However, gas conductance at full load consists of not only the convective component but also the radiative component. The radiative component of the gas conductance can be calculated by the equation : (Ref. [8])

$$G_{ab,r} = \frac{\sigma\ \overline{AB}\ (T_a^{\ 4} - T_b^{\ 4})}{T_a - T_b} \tag{13}$$

where σ is the Stefan-Boltzmann constant, T_a and T_b are temperatures of the furnace gas and heat exchanger wall, respectively; and $\overline{AB}$ is the radiative flux area which is a function of surface area A_b, gas emissivity ε_a and wall emissivity ε_b, namely:

$$\overline{AB} = \frac{A_b}{\frac{1}{\varepsilon_a} + \frac{1}{\varepsilon_b} - 1} \tag{14}$$

The gas emissivity ε_a for the combustion products can be calculated by equation[2]: (Ref. [8])

$$\varepsilon_a = 258\ \frac{(L_e P_{ch})^{0.38}}{T_a} \qquad \text{(for } P_{ch} > 0.2 \text{ atm.)}$$

$$\text{or} \quad \varepsilon_a = 529\ \frac{(L_e P_{ch})^{0.64}}{T_a} \qquad \text{(for } P_{ch} < 0.2 \text{ atm.)} \tag{15}$$

where P_{ch} is the sum of partial pressures (in atmosphere) of CO_2 and H_2O in combustion products, equivalent radiation path length (in meters) is given by the equation: (Ref. [9])

$$L_e = 1.85S \tag{16}$$

and S is the spacing of the gas flow passages. With the radiative component of conductance $G_{ab,r}$ calculated by equation 13, the convective conductance $G_{ab,c}$ at full load can now be obtained from

$$G_{ab,c} = G_{ab,f} - G_{ab,r} \tag{17}$$

As the partial pressure of H_2O and CO_2 in draft air during the off cycle is negligible, we need only to consider the convective conductance while the burner is off. For turbulent convection, the convective conductance [7] is proportional to $(\dot{m}_a^{0.8} \times T_a^{0.15})$. The off-cycle gas conductance is related to the convective conductance at full load by the equation:

$$G_{ab,off} = \left(\frac{T_{a,off}}{T_{a,f}}\right)^{0.15} \left(\frac{\dot{m}_{a,off}}{\dot{m}_{a,f}}\right)^{0.8} G_{ab,c} \tag{18}$$

With T_a obtained from the equation 1 set, $(\dot{m}_{a,off}/\dot{m}_{a,f})$ from equation 11, the off-cycle $G_{ab,off}$ can be calculated by equation 18.

Finally, conductance $(G_{bc,off})$ and capacity rate $(\dot{C}_{c,off})$ for circulating air, while the blower is off, are calculated by the theory of free convection [10]; using properties of air at average temperature 50C yields the following simplified equations:

$$G_{bc,off} = 3{,}750\ A_b \left(\frac{T_{b,off} - T_{c,off}}{L_b}\right)^{0.25} \tag{19}$$

$$\dot{C}_{c,off} = 12{,}400\ A_b \left(\frac{T_{b,off} - T_{c,off}}{L_b}\right)^{0.25} \tag{20}$$

where A_b, $T_{b,off}$, $T_{c,off}$ and L_b are in meter and degree Celsius and $G_{bc,off}$ and $C_{c,off}$ are in joules per hour per degree Celsius.

The formulation in the above three subsections completes a mathematical model for the furnace simulation, using the equation 1 set with coefficients and initial and boundary conditions described above. Organization of the computer program for solution of the equation 1 set is described in the following subsection.

Numerical Procedure and Computer Program

In order to solve the simulation equation 1 set by computer, differential equations must first be written in finite difference form. Derivation of finite difference equations can be started by

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[2] Equation 15 is valid for non-luminous gas burners; certain modifications are required for luminous gas burners or oil burners.

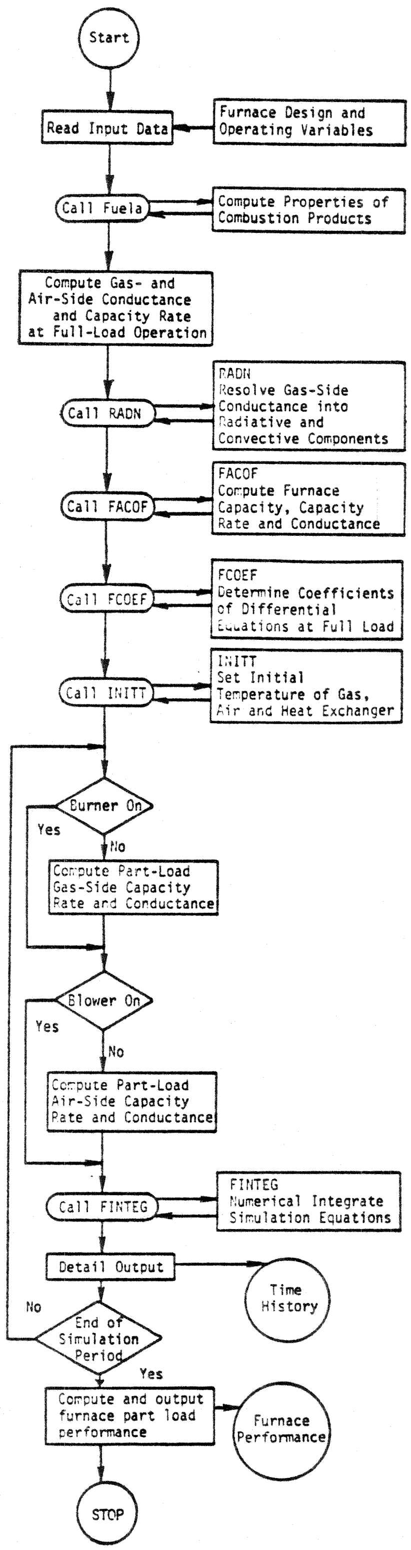

Figure 4. Flow Chart for Furnace Simulation Program DEPAF

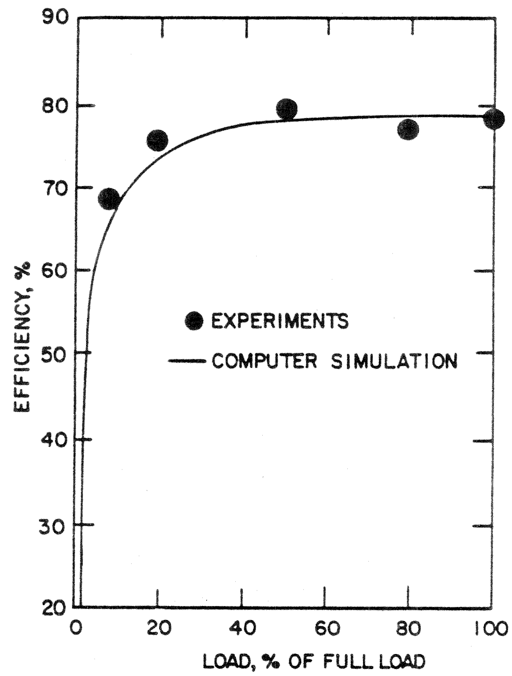

Figure 5. Predicted and Measured Part-Load Performance of Furnace (Blower: Continuous Operation)

furnace. The computer was run under the same conditions as the experiments. Excellent agreement between the theoretical prediction and experiment can be observed. It is to be noted that the efficiency η_ℓ plotted in Figure 5 is for the case of continuous-blower operation and obtained by dividing the furnace gross output (i.e., from the integral of the product of circulating-air temperature rise across the furnace and mass flow rate) by the total fuel input. Computer simulation has predicted accurately not only the part-load efficiency η_ℓ but also the time history of furnace temperatures, as can be seen in Figures 6 and 7, where temperatures of circulating air leaving the furnace are plotted versus time. Figure 6 is for the case with burner operating at 80% on-time and 20% off-time and with blower in continuous operation. Figure 8 is for the case with the burner on 20% of the time and off 80%, and with the blower on at supply temperature 58C and off at 38C. These comparisons illustrate that satisfactory prediction of furnace performance can be obtained by computer simulation.

Energy-Saving Measures

There is much interest in designing and operating residential furnaces to minimize heating energy consumption and operating cost. The effectiveness of proposed energy-saving measures for furnaces can be examined by the present simulation program. Hundreds of computer runs, using DEPAF, have been made for a variety of design, operating, and weather conditions. Space does not allow a presentation of complete results in a single paper. However, several examples will be given below.

Laboratory tests of furnaces are usually based upon either the exhaust loss or input and output measurements. In a residential heating system, air enters the furnace and draft diverter at the furnace

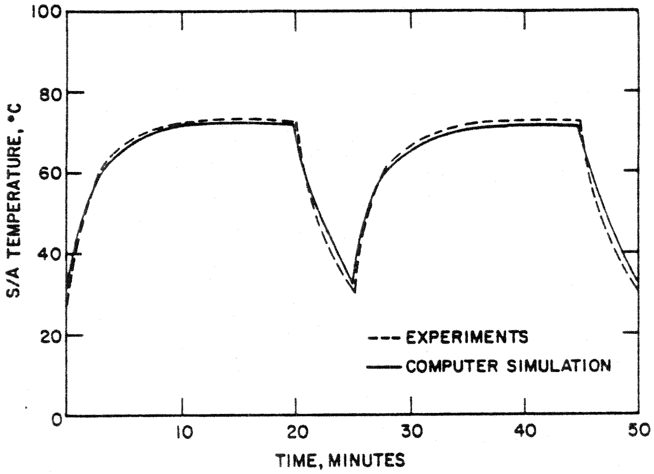

Figure 6. Predicted and Measured Supply Air Temperature Versus Time (Burner: 80% On-Time, 20% Off-Time; Blower: Continuous Operation)

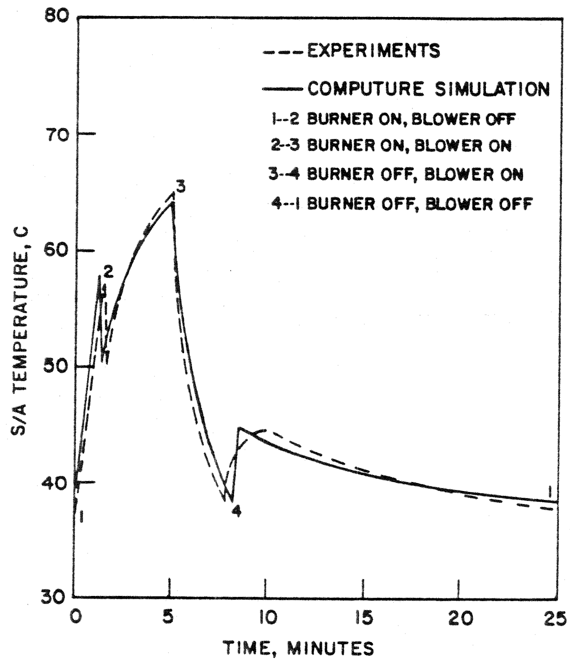

Figure 7. Predicted and Measured Supply Air Temperature Versus Time (Burner: 20% On-Time, 80% Off-Time; Blower: On at 58C, Off at 38C)

room temperature. This air has in fact been heated from the outdoor to the furnace room temperature. Useful heat available for heating a home is therefore less than the laboratory-measured efficiency. The heat spent in heating the furnace air (including combustion, draft and relief air) from the outdoor to the furnace room temperature must be accounted for. This heat should not, however, be charged completely against the furnace, because the presence of the stack reduces the furnace room pressure which tends to decrease the normal structure exfiltration. Janssen and Bonne [11] indicated that the average value for the infiltration is 0.7 (i.e., on the average, 70% of furnace exfiltration is chargeable

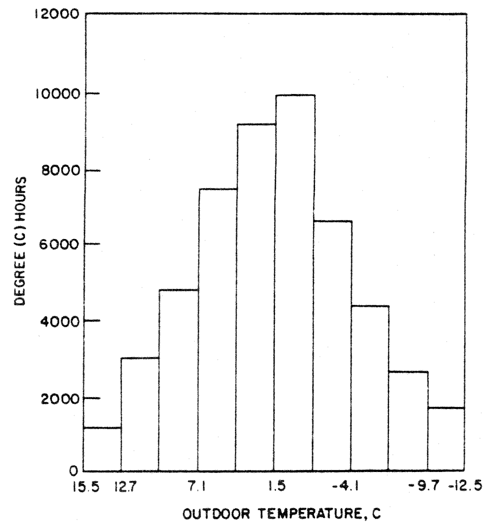

Figure 8. Ten-Year Average Distribution of Degree-C Hours for Washington, D. C.

against the furnace). Field investigations are being undertaken by several research groups to determine the infiltration parameters under different conditions. In the following examples the infiltration value 0.7 will be used throughout.

Let us now consider a reference residential furnace with atmospheric burner having full-load input rate 30 kW, a pilot fuel supply 0.3 kW, a blower electric input 0.5 kW. The furnace uses natural gas with 61% excess air, has steady-state full-load efficiency (excluding infiltration loss) 78% and has 100% relief air. The residence is assumed to be maintained at 20.5C, to have an internal source equivalent at a ΔT of 5.5C, and a structure load of 0.32 kW/C. The thermostat controls are such that the burner operates at 6 cycles per hour at half load and the blower comes on at supply air temperature of 60C and off at 43C. DEPAF has been run for the above-described system using Washington, D. C. weather data shown in Figure 8 [12].

In addition, DEPAF has been run with several design changes incorporated into the above-described reference system. These changes include: 1) use of intermittent electric ignition, 2) use of automatic stack damper, 3) use of intermittent ignition and power burner, and 4) use of intermittent ignition and automatic stack damper. Results of these runs, together with the run for the reference system in terms of fuel utilization efficiency η_u (including infiltration loss), are plotted in Figure 9 versus outdoor temperature.

The combination of Figures 8 and 9, remembering the residence structure load is 0.32 kW/C, yields annual performance for the systems described above. Table 1 shows the calculated annual fuel consumption and efficiency for these furnace systems. Also shown in this table are electric consumptions and savings in fuel, electricity and operating cost, when the energy-saving measures discussed above are adopted. It can be seen in this table that savings in operating cost and improvement in annual efficiency may be considerable through appropriate selection of design variables for the furnace.

Table 1 Gas-Fired Central Forced-Warm-Air Furnace, Annual Performance (NBS/DEPAF)

Run No.	System Parameters	Electric* kWH/Yr.	\$/Yr.	Save%	Fuel* Jx10^9/Yr.	\$/Yr.	Save %	Utilization Efficiency	Operating Cost \$/Yr.	Save %
1	Reference	490	20	---	88	226	---	64	246	---
2	Intermittent Ignition	464	19	5	82	211	7	69	229	7
3	Automatic Stack Damper, 85% Closure at Off-Cycle	463	19	6	78	201	11	72	219	11
4	Intermittent Ignition Power Burner	553	22	-13	75	192	15	76	214	12
5	Intermittent Ignition Automatic Stack Damper, 85% Closure at Off-Cycle	437	17	11	74	186	17	77	20~~6~~3	16

*Electric Rate \$0.04 per Kwh, Gas Rate \$2.56/10^9J.

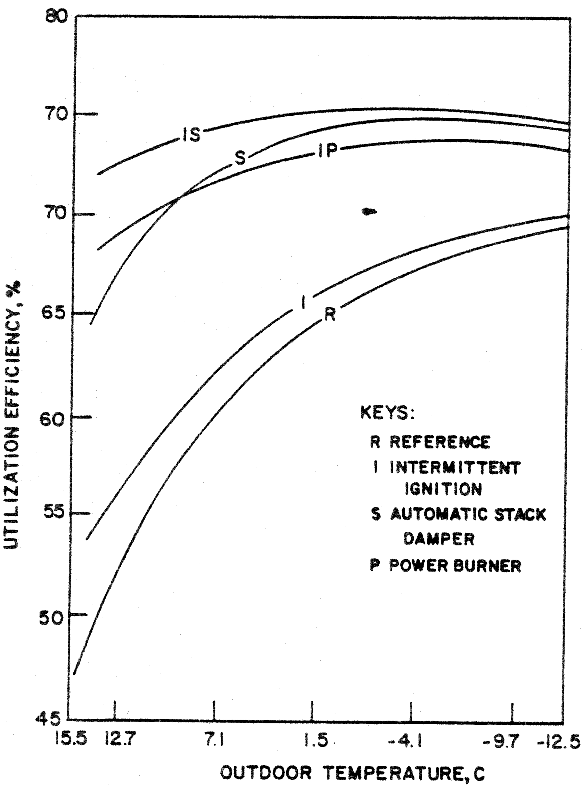

Figure 9. Fuel Utilization Efficiencies of Furnace Systems Versus Outdoor Temperature

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